Winter term 2026/27
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Presence sheet 4

Mathematics for Machine Learning

Tutorial of **Week 5 (09. - 13.11.2026)**

Exercise 1 (**Positive semi-definite matrices**)**.** Find a matrix that is not positive semi-definite but has only positive entries.

Exercise 2 (**Matrix norms**)**.**
Consider the matrix $A = \begin{pmatrix} 1 & 2 \\ -1 & 2 \end{pmatrix}$ and calculate the following norms:

a) Supremum norm: $\|A\|_{\max}$

b) Frobenius norm: $\|A\|_F$

c) Spectral norm: $\|A\|_2$

d) Nuclear norm: $\|A\|_*$

Exercise 3 (**Frobenius norm**)**.**
Consider the Frobenius norm $\|A\|_F = \sqrt{\sum_{i,j} a_{ij}^2}$.

1. Show that this is a norm on the space $\mathbb{R}^{n\times n}$.
2. Show that $\|A\|_F = \sqrt{\mathrm{tr}(A^T A)}$, where

$$\mathrm{tr}(M) = M_{11} + ... + M_{nn}$$

is the trace of a matrix $M \in \mathbb{R}^{n\times n}$, defined as the sum of its diagonal entries.

Exercise 4 (**Examples**)**.**
Find an example of (or explain why such an example cannot exist)

1. A square matrix with zero as its only eigenvalue and 5 as its only singular value.
2. A square matrix with zero as its only eigenvalue and singular values 0 and 5.
3. A square matrix $A \neq 0$ whose singular values all equal zero.

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