Winter term 2026/27
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# Presence sheet 01

## Mathematics for Machine Learning

Tutorial of **Week 02 (19. - 23.10.2026)**

**Exercise 1** (**Diagonal Matrix).**
Consider the diagonal matrix

$$A = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}. \tag{1}$$

a) Calculate $A^2$.

b) Calculate $A^n$ for any $n \in \mathbb{N} \setminus \{0\}$.

**Exercise 2** (**Basis).**
Consider the following sets of vectors in $\mathbb{R}^3$. Decide which of them are bases of $\mathbb{R}^3$.

a) $\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\} \subset \mathbb{R}^3$

b) $\left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} \right\} \subset \mathbb{R}^3$

c) $\left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right\} \subset \mathbb{R}^3$

d) $\left\{ \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \right\} \subset \mathbb{R}^3$

e) $\left\{ \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \right\} \subset \mathbb{R}^3$

**Exercise 3** (**Basis and Dimension).**
Consider the subset $S := \{(x, y, z) \in \mathbb{R}^3 | x + y + z = 0\} \subset \mathbb{R}^3$.

a) Is $S$ a subspace of $\mathbb{R}^3$?

b) Find a basis of $S$.

c) What is the dimension of this subspace?

**Exercise 4** (**Linear maps).**
Decide whether the following maps are linear:

a) $T : \mathbb{R} \to \mathbb{R}$ with $T(x) = x^2$

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b) $T : \mathbb{R}^3 \to \mathbb{R}$ with $T((x, y, z)^t) = 3x - y + z$

c) $T : \mathbb{R}^2 \to \mathbb{R}^2$ with $T((x, y)^t) = (x + y, x - y)^t$

**Exercise 5** (**Range and kernel).**
Consider the matrices

$$A = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix} \qquad \text{and} \qquad B = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}.$$

a) For both matrices determine the kernel and the range and give a basis of each.

b) What is the relation of $\dim \ker$, $\dim \text{range}$ and the number of columns.

**Exercise 6** (**Projection matrix).**
Consider the matrix

$$P = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}.$$

a) Compute $P^2$. What do you observe and what is the meaning of it geometrically?

b) Determine $\ker(P)$ and $\text{range}(P)$ and give a basis of each.

c) Compute $Pv$ for $v = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$. Sketch $\text{range}(P)$, $\ker(P)$, the vector $v$ and the vector $Pv$ in the plane. What does $P$ do geometrically?